INTRODUCTION TO MATHCAD
Differential Calculus

In this lesson you will earn how to use Mathcad to numerically differentiate equations. For this EM375 course, partial differentiation is an essential calculation for, for example, error analysis and the propagation of errors.

Symbolic differentiation is not part of this tutorial.

Open a new worksheet in preparation for this lesson.

DIFFERENTIATING A FUNCTION:

As a first example, we will consider the displacement of a particle (do you remember particle kinematics from EM232 Dynamics?). Let's assume the displacement (in meters) of a particle is given by:

$$s = 30t^2 - 1.45t^3$$

We will be differentiating this equation with respect to time, t (seconds) to find the velocity, and then the acceleration. Do this differentiation by hand yourself, and show you can get (without using Mathcad):

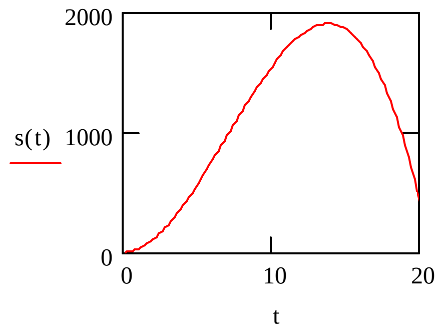
$$v = 60t - 4.35t^2$$
$$a = 60 - 8.70t$$

Let's get the displacement function into your worksheet:

s(t):30*t^2® -1.45*t^3

When you plot this function, you can let Mathcad "choose" a range for "*t*", but it is better to define it yourself.

Define *t* to be a range variable from zero to 20 in steps of 0.2. Then plot a graph of *s(t)* vs. *t*.

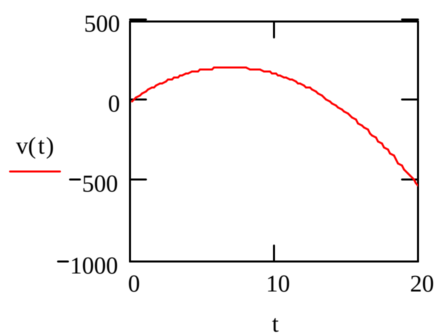


SELECTING THE DIFFERENTIAL OPERATOR

We will now differentiate this displacement function to get velocity. Click a blank area of your worksheet (after the graph). You can enter the differential operator in either of two ways. Either use the keyboard shortcut **?** (question mark), or select the d/dt option from the Calculus template. The d/dt will appear with empty placeholders. We are differentiating position to get velocity, so fill in the placeholders appropriately to get:

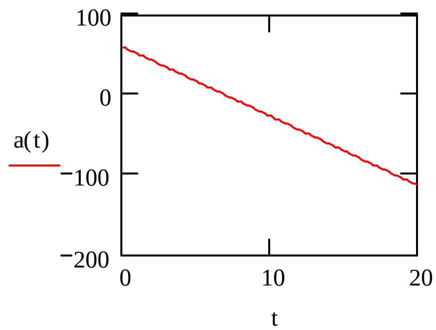
$$\boxed{v(t) := \frac{d}{dt}s(t)}$$

You now have differentiated position, *s*, and put the result into variable *v*. Plot *v(t)* vs. *t* to see the velocity graph.



Finally, let's look at the acceleration, which is found by differentiating *v(t)*.

$$a(t) := \frac{d}{dt}v(t)$$

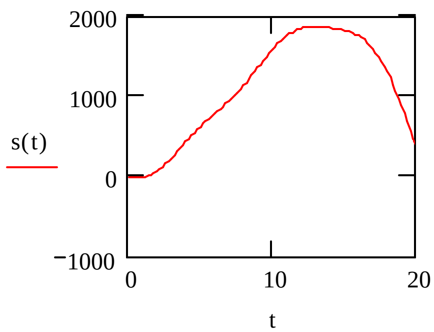


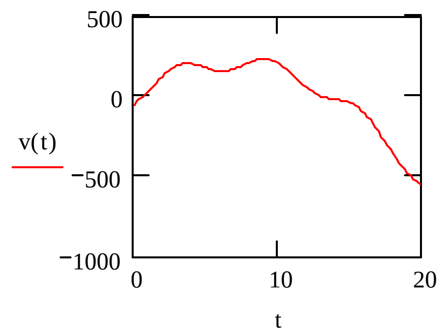
Notice that the acceleration is a linear function of time.

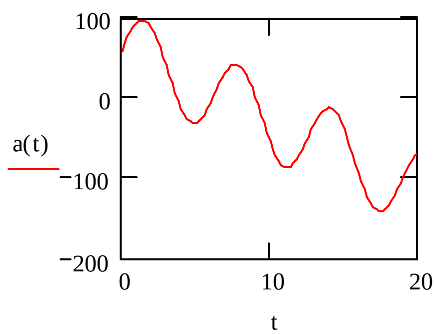
Go back to your displacement function and edit it to be as follows:

$$\boxed{s(t) := 30 \cdot t^2 - 1.45 \cdot t^3 - 50 \sin(t)}$$

You should get the following displacement, velocity and acceleration graphs:







<u>AN ERROR PROPAGATION EXAMPLE</u>

For this example we will consider calculating the polar second moment of inertia for a hollow cylinder. The theoretical equation is:

$$J = \frac{p}{64}\left(D_O^4 - D_I^4\right)$$

For this example, let's assume the inside and outside diameters are measured as D_O = 6.25 ± 0.02 ins and D_I = 5.00 ± 0.03 ins respectively (all uncertainties are at the 95% level). The question is, what is J and its uncertainty?

Before we turn to Mathcad, we will develop the solution analytically. That way we can check Mathcad's output. The uncertainty in J, w_J, is determined as follows:

$$w_J = \left\{\left(w_{DO}\frac{\partial J}{\partial D_O}\right)^2 + \left(w_{DI}\frac{\partial J}{\partial D_I}\right)^2\right\}^{1/2}$$

where w_{DO} = 0.02 = uncertainty in outside diameter, and w_{DI} = 0.03 = uncertainty in inside diameter. Solving for J and w_J, we get:

J = 44.2217 in^4
w_J = 1.209 in^4

In other words, the polar moment of inertia is 44.2217 ± 1.209 in^4.

Now let's turn to Mathcad.

First, define a function for J, and define the diameters and their uncertainties:

$\mathrm{Do} := 6.25 \qquad \mathrm{Di} := 5.0$

$\mathrm{wDO} := 0.02 \qquad \mathrm{wDI} := 0.03$

$$\mathrm{J(Do, Di)} := \pi\cdot\frac{\left(\mathrm{Do}^4 - \mathrm{Di}^4\right)}{64} \qquad \mathrm{J(Do, Di)} = 44.222$$

Now use the differential operator to evaluate the uncertainty:

$$\mathrm{wJ} := \left[\left(\mathrm{wDO}\cdot\frac{d}{d\mathrm{Do}}\mathrm{J(Do, Di)}\right)^2 + \left(\mathrm{wDI}\cdot\frac{d}{d\mathrm{Di}}\mathrm{J(Do, Di)}\right)^2\right]^{0.5}$$

$\mathrm{wJ} = 1.209$

Once we consider units, we get the same answers as for the analytical solution.